Online Appendix to

When Do Times of Increasing Uncertainty Call for Centralized Harmonization in International Policy Coordination?

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Part A

Proof of Claim 1: $\theta_1$ and $\theta_2$ are private information of jurisdictions 1 and 2, respectively. The appropriate equilibrium concept is thus Bayesian Nash equilibrium, where jurisdictions' equilibrium strategies are functions of their respective fundamentals, or 'types'. Given jurisdiction 2's equilibrium policy choice $p_2(\theta_2)$, jurisdiction 1 chooses $p_1$ to maximize

$$-\gamma_1(\theta_1-p_1)^2 - (1-\gamma_1)E(p_1-p_2(\theta_2))^2,$$

where $E$ is the expectation operator. Obtaining and re-arranging the corresponding first-order condition gives

$$p_1 = \gamma_1 \theta_1 + (1-\gamma_1)E p_2(\theta_2).$$

(A2)

Recognizing that jurisdiction 1's equilibrium strategy is a function of $\theta_1$ and taking expectation with respect to $\theta_1$ on both sides of (A2) yields

$$E p_1(\theta_1) = (1-\gamma_1)E p_2(\theta_2).$$

(A3)

Following analogous steps as above to characterize jurisdiction 2's equilibrium policy choice gives

$$p_2 = \gamma_2 \theta_2 + (1-\gamma_2)E p_1(\theta_1)$$

and, eventually,

$$E p_2(\theta_2) = (1-\gamma_2)E p_1(\theta_1).$$

(A5)

Given $\gamma_1, \gamma_2 \in (0,1)$, the system of equations (A5) and (A3) has a unique solution: $E p_1(\theta_1) = 0$ and $E p_2(\theta_2) = 0$. Thus, from (A2) and (A4), $p_1^D = \gamma_1 \theta_1$ and $p_2^D = \gamma_2 \theta_2$. □

Proof of Claim 2: The choice of $p$ to maximize $E\{-\omega_1 \gamma_1 (\theta_1-p)^2 - \omega_2 \gamma_2 (\theta_2-p)^2\}$ yields the first-order condition $-\omega_1 2 \gamma_1 E(\theta_1-p)(-1)-\omega_2 2 \gamma_2 E(\theta_2-p)(-1)=0$, which, when simplified, gives $p[\omega_1 \gamma_1 + \omega_2 \gamma_2]=0$. Because $\omega_1 \gamma_1 + \omega_2 \gamma_2 > 0$, $p = 0$. With the objective function concave in $p$, the second-order sufficient condition for a maximum is satisfied. □

Derivation of expressions (2a), (2b), (3), (4a), (4b), (5), (6), and (7): Follows from straightforward algebra using the properties of $\theta_1$ and $\theta_2$: $E(\theta_1)=E(\theta_2)=0$, $E(\theta_1^2)=\sigma_1^2$, $E(\theta_2^2)=\sigma_2^2$, and, because $\theta_1$ and $\theta_2$ are independent, $E(\theta_1 \theta_2)=0$. □

Proof of Proposition: Suppose that $\nu_1>0$. (The proof for the case when $\nu_2>0$ is analogous and thus omitted.) Then, because $\nu_1+\nu_2<0$, we know that $\nu_2<0$. From (7), therefore, $d\Delta>0$ if and only if $d\sigma_2^2/d\sigma_1^2 > \xi$, where $\xi = -(\gamma_2^2 \nu_2)/(\gamma_1^2 \nu_1)$>0. To prove the properties of $\xi$, let $s=\omega_2/\omega_1$. Note that $\nu_2>0$ implies $s>1$. Then, since $\nu_1=\omega_2(1-\gamma_1)/\omega_1$, $\nu_1>0$ may be expressed as

$$s(1-\gamma_2)>1.$$  

(A6)

Also, $\xi$ can be written as

$$\xi = \frac{\gamma_2^2 s(1-\gamma_1)}{\gamma_1^2 s(1-\gamma_2)-1}. $$

(A7)
From (A7), it follows that, first,
\[
\frac{\partial \xi}{\partial s} = \frac{\gamma_1^2 \gamma_2^2 [(1 - \gamma_2)(1 - \gamma_1) - 1]}{\gamma_1^4 s (1 - \gamma_2)^2},
\]
which is negative given that \(\gamma_1, \gamma_2 \in (0, 1)\). Second,
\[
\frac{\partial \xi}{\partial \gamma_1} = \frac{\gamma_1 \gamma_2^2 [s (1 - \gamma_2) - 1] [-\gamma_1 - 2(s - 1)]}{\gamma_1^4 s (1 - \gamma_2)^2},
\]
which is negative by (A6) and since \(s > 1\). Third,
\[
\frac{\partial \xi}{\partial \gamma_2} = \frac{2\gamma_2 [s - (1 - \gamma_1)] \gamma_1^2 [s (1 - \gamma_2) - 1] + \gamma_2^2 [s - (1 - \gamma_1)] \gamma_1^2 s}{\gamma_1^4 s (1 - \gamma_2)^2},
\]
which is positive by (A6) and because \(s > 1\).

Part B

This appendix analyzes the model as developed in Sections 2 and 3 of the paper while relaxing the assumption that the means of \(\theta_1\) and \(\theta_2\) respectively equal to zero. Specifically, assume that now \(E \theta_1 = \mu_1 \neq 0\) and \(E \theta_2 = \mu_2 \neq 0\), with \(\mu_1 \neq \mu_2\). Thus, \(\sigma_1^2 = E (\theta_1 - \mu_1)^2 = E \theta_1^2 - \mu_1^2\) and \(\sigma_2^2 = E (\theta_2 - \mu_2)^2 = E \theta_2^2 - \mu_2^2\). All other assumptions adopted in Sections 2 and 3 continue to hold.

Consider, first, decentralized (anarchic) policy-making. Jurisdiction 1 chooses \(p_1\) to maximize (A1), giving rise to (A2). Recognizing that jurisdiction 1’s equilibrium strategy is a function of \(\theta_1\) and taking expectation with respect to \(\theta_1\) on both sides of (A2) yields:
\[
\gamma_1 \mu_1 + (1 - \gamma_1) E p_2 (\theta_2) = E p_1 (\theta_1).
\]
By symmetry,
\[
\gamma_2 \mu_2 + (1 - \gamma_2) E p_1 (\theta_1) = E p_2 (\theta_2).
\]
Solving the systems of equations (B1) and (B2) for \(E p_1 (\theta_1)\) and \(E p_2 (\theta_2)\) and plugging the resulting expressions into (B1) and (B2) gives
\[
p^D_1 = \gamma_1 \theta_1 + (1 - \gamma_1) \frac{\gamma_2 \mu_2 + (1 - \gamma_2) \gamma_1 \mu_1}{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2},
\]
\[
p^D_2 = \gamma_2 \theta_2 + (1 - \gamma_2) \frac{\gamma_1 \mu_1 + (1 - \gamma_1) \gamma_2 \mu_2}{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2}.
\]
Observe that if \(\mu_1 = \mu_2 = 0\), (B3) and (B4) reduce to \(p^D_1 = \gamma_1 \theta_1\) and \(p^D_2 = \gamma_2 \theta_2\), as stipulated in Claim 1 in the paper. Then,
\[
EW^D = E \{ -\omega_1 \gamma_1 (\theta_1 - p^D_1)^2 - \omega_1 (1 - \gamma_1) (p^D_1 - p^D_2)^2 - \omega_2 \gamma_2 (\theta_2 - p^D_2)^2 - \omega_2 (1 - \gamma_2) (p^D_2 - p^D_1)^2 \},
\]
where
\[
\theta_1 - p^D_1 = (1 - \gamma_1) \theta_1 - (1 - \gamma_1) \frac{\gamma_2 \mu_2 + (1 - \gamma_2) \gamma_1 \mu_1}{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2}.
\]
\[ \theta_2 - p_2^D = (1 - \gamma_2) \theta_2 - (1 - \gamma_2) \frac{\gamma_1 \mu_1 + (1 - \gamma_1) \gamma_2 \mu_2}{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2} \quad (B7) \]

\[ p_1^D - p_2^D = \gamma_1 \theta_1 - \gamma_2 \theta_2 + \frac{(1 - \gamma_1) \gamma_2^2 \mu_2 - (1 - \gamma_2) \gamma_1^2 \mu_1}{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2} \quad (B8) \]

Inserting (B6)-(B8) into (B5), taking the expectation, noting that \( E\theta_1^2 = \sigma_1^2 + \mu_1^2 \), \( E\theta_2^2 = \sigma_2^2 + \mu_2^2 \), and collecting terms, it follows that

\[ EW^D = -\left[ \omega_1 (1 - \gamma_1) \gamma_1 + \omega_2 (1 - \gamma_2) \gamma_2^2 \right] \sigma_1^2 - \left[ \omega_2 (1 - \gamma_2) \gamma_2 + \omega_1 (1 - \gamma_1) \gamma_1^2 \right] \sigma_2^2 + J, \quad (B9) \]

where the term \( J = J(\gamma_1, \gamma_2, \omega_1, \omega_2, \mu_1, \mu_2) \) is independent of \( \sigma_1^2 \) and \( \sigma_2^2 \). Observe that (B9) differs from the expression (3) in the paper by the term \( J \) only.

Consider, next, centralized harmonization of policymaking. Acting ex ante, before \( \theta_1 \) and \( \theta_2 \) are realized, the supra-jurisdictional authority chooses a uniform policy \( p \) to maximize

\[ E \left\{ -\omega_1 \gamma_1 \left( \theta_1 - p \right)^2 - \omega_2 \gamma_2 \left( \theta_2 - p \right)^2 \right\}. \quad (B10) \]

Obtaining and re-arranging the resulting first-order condition (second-order condition for a maximum is satisfied) gives

\[ p^H = \frac{\omega_1 \gamma_1 \mu_1 + \omega_2 \gamma_2 \mu_2}{\omega_1 \gamma_1 + \omega_2 \gamma_2} \quad (B11) \]

Note that if \( \mu_1 = \mu_2 = 0 \), (B11) simplifies to \( p^H = 0 \), as noted in Claim 2 in the paper. Then,

\[ EW^H = E \left\{ -\omega_1 \gamma_1 \left( \theta_1 - p^H \right)^2 - \omega_2 \gamma_2 \left( \theta_2 - p^H \right)^2 \right\}, \quad (B12) \]

where

\[ \theta_1 - p^H = \frac{\omega_1 \gamma_1 \mu_1 + \omega_2 \gamma_2 \mu_2}{\omega_1 \gamma_1 + \omega_2 \gamma_2} \quad (B13) \]

\[ \theta_2 - p^H = \frac{\omega_1 \gamma_1 \mu_1 + \omega_2 \gamma_2 \mu_2}{\omega_1 \gamma_1 + \omega_2 \gamma_2} \quad (B14) \]

Inserting (B13) and (B14) into (B12), taking the expectation, noting that \( E\theta_1^2 = \sigma_1^2 + \mu_1^2 \), \( E\theta_2^2 = \sigma_2^2 + \mu_2^2 \), and collecting terms, it follows that

\[ EW^H = -\omega_1 \gamma_1 \sigma_1^2 - \omega_2 \gamma_2 \sigma_2^2 + K, \quad (B15) \]

where the term \( K = K(\gamma_1, \gamma_2, \omega_1, \omega_2, \mu_1, \mu_2) \) is independent of \( \sigma_1^2 \) and \( \sigma_2^2 \). Observe that (B15) and the expression (5) in the paper differ only by the term \( K \).

Comparison of (B15) and (B9) yields

\[ \Delta \equiv EW^H - EW^D = \nu_1 \gamma_1 \sigma_1^2 + \nu_2 \gamma_2 \sigma_2^2 + K - J, \quad (B16) \]

where \( \nu_1 \equiv \omega_2 (1 - \gamma_2) - \omega_1 \) and \( \nu_2 \equiv \omega_1 (1 - \gamma_1) - \omega_2 \) (see Section 3 of the paper). Letting \( d\sigma_1^2, \ d\sigma_2^2 > 0 \),

\[ d\Delta = \gamma_1^2 \nu_1 d\sigma_1^2 + \gamma_2^2 \nu_2 d\sigma_2^2, \quad (B17) \]

which coincides with expression (7) in the paper. Thus, when \( \mu_1 \neq \mu_2 \neq 0 \), the Proposition stated in Section 3 continues to hold. \( \square \)
Part C

This appendix shows that under the assumptions of the model as specified in Sections 2 and 3 of the paper, policy harmonization emerges endogenously under the centralized, top-down approach: acting ex ante, the supra-jurisdictional authority chooses a uniform policy \( p_1 = p_2 = p \) even if mandating diverse policies in different jurisdictions (i.e. \( p_1 \neq p_2 \)) is a possibility.

Consider the following problem: acting ex ante, before \( \theta_1 \) and \( \theta_2 \) are realized, the supra-jurisdictional authority chooses \( p_1 \) and \( p_2 \) to maximize

\[
E\left\{-\omega_1 \gamma_1 (\theta_1 - p_1)^2 - \omega_1 (1- \gamma_1) (p_1 - p_2)^2 - \omega_2 \gamma_2 (\theta_2 - p_2)^2 - \omega_2 (1- \gamma_2) (p_2 - p_1)^2 \right\}
\]

(C1)

Obtaining and re-arranging the resulting first-order necessary conditions (second-order conditions for a maximum are satisfied) gives

\[
[\omega_1 + \omega_2 (1- \gamma_2)] p_1 - [\omega_1 (1- \gamma_1) + \omega_2 (1- \gamma_2)] p_2 = 0
\]

(C2)

\[
[\omega_1 (1- \gamma_1) + \omega_2 (1- \gamma_2)] p_1 - [\omega_2 + \omega_1 (1- \gamma_1)] p_2 = 0
\]

(C3)

Equations (C2) and (C3) constitute a linear homogenous system \( AX = 0 \), where \( X = [p_1 \ p_2]' \) and the matrix of coefficients equals

\[
A = \begin{bmatrix}
\omega_1 + \omega_2 (1- \gamma_2) & -[\omega_1 (1- \gamma_1) + \omega_2 (1- \gamma_2)] \\
[\omega_1 (1- \gamma_1) + \omega_2 (1- \gamma_2)] & -[\omega_2 + \omega_1 (1- \gamma_1)]
\end{bmatrix}
\]

(C4)

From (C4), it follows that the determinant of \( A \) equals

\[
\det(A) = \omega_1 \omega_2 (\gamma_1 \gamma_2 - \gamma_1 - \gamma_2) - \omega_2^2 (1- \gamma_2) \gamma_2 - \omega_1^2 (1- \gamma_1) \gamma_1 \neq 0.
\]

(C5)

(C5) implies \( p_1 = p_2 = 0 \). \( \square \)