Online Appendix to

When Do Times of Increasing Uncertainty Call for Centralized Harmonization in International Policy Coordination?

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Part A

Proof of Claim 1: θ_1 and θ_2 are private information of jurisdictions 1 and 2, respectively. The appropriate equilibrium concept is thus Bayesian Nash equilibrium, where jurisdictions' equilibrium strategies are functions of their respective fundamentals, or 'types'. Given jurisdiction 2's equilibrium policy choice $p_2(\theta_2)$, jurisdiction 1 chooses p_1 to maximize

$$-\gamma_1(\theta_1-p_1)^2-(1-\gamma_1)\text{E}(p_1-p_2(\theta_2))^2, \tag{A1}$$

where E is the expectation operator. Obtaining and re-arranging the corresponding first-order condition gives

$$p_1=\gamma_1\theta_1+(1-\gamma_1)\text{E}p_2(\theta_2). \tag{A2}$$

Recognizing that jurisdiction 1's equilibrium strategy is a function of θ_1 and taking expectation with respect to θ_1 on both sides of (A2) yields

$$\text{E}p_1(\theta_1)=(1-\gamma_1)\text{E}p_2(\theta_2). \tag{A3}$$

Following analogous steps as above to characterize jurisdiction 2's equilibrium policy choice gives

$$p_2=\gamma_2\theta_2+(1-\gamma_2)\text{E}p_1(\theta_1) \tag{A4}$$

and, eventually,

$$\text{E}p_2(\theta_2)=(1-\gamma_2)\text{E}p_1(\theta_1). \tag{A5}$$

Given γ_1, $\gamma_2\in(0,1)$, the system of equations (A5) and (A3) has a unique solution: $\text{E}p_1(\theta_1)=0$ and $\text{E}p_2(\theta_2)=0$. Thus, from (A2) and (A4), $p_1^D=\gamma_1\theta_1$ and $p_2^D=\gamma_2\theta_2$. □

Proof of Claim 2: The choice of p to maximize $\text{E}\{-\omega_1\gamma_1(\theta_1-p)^2-\omega_2\gamma_2(\theta_2-p)^2\}$ yields the first-order condition $-\omega_1 2\gamma_1\text{E}(\theta_1-p)(-1)-\omega_2 2\gamma_2\text{E}(\theta_2-p)(-1)=0$, which, when simplified, gives $p[\omega_1\gamma_1+\omega_2\gamma_2]=0$. Because $\omega_1\gamma_1+\omega_2\gamma_2>0$, $p^H=0$. With the objective function concave in p, the second-order sufficient condition for a maximum is satisfied. □

Derivation of expressions (2a), (2b), (3), (4a), (4b), (5), (6), and (7): Follows from straightforward algebra using the properties of θ_1 and θ_2: $\text{E}(\theta_1)=\text{E}(\theta_2)=0$, $\text{E}(\theta_1^2)=\sigma_1^2$, $\text{E}(\theta_2^2)=\sigma_2^2$, and, because θ_1 and θ_2 are independent, $\text{E}(\theta_1\theta_2)=0$. □

Proof of Proposition: Suppose that $\nu_1>0$. (The proof for the case when $\nu_2>0$ is analogous and thus omitted.) Then, because $\nu_1+\nu_2<0$, we know that $\nu_2<0$. From (7), therefore, $\text{d}\Delta>0$ if and only if $\text{d}\sigma_2^2/\text{d}\sigma_1^2>\xi$, where $\xi=-(\gamma_2^2\nu_2)/(\gamma_1^2\nu_1)>0$. To prove the properties of ξ, let $s=(\omega_2/\omega_1)$. Note that $\nu_2>0$ implies $s>1$. Then, since $\nu_1=\omega_2(1-\gamma_1)-\omega_1$, $\nu_1>0$ may be expressed as

$$s(1-\gamma_2)>1. \tag{A6}$$

Also, ξ can be written as

$$\xi=\frac{\gamma_2^2[s-(1-\gamma_1)]}{\gamma_1^2[s(1-\gamma_2)-1]}. \tag{A7}$$

From (A7), it follows that, first,

$$\frac{\partial \xi}{\partial s}=\frac{\gamma_1^2\gamma_2^2[(1-\gamma_2)(1-\gamma_1)-1]}{\gamma_1^4[s(1-\gamma_2)-1]^2}, \tag{A8}$$

which is negative given that $\gamma_1,\gamma_2\in(0,1)$. Second,

$$\frac{\partial \xi}{\partial \gamma_1}=\frac{\gamma_1\gamma_2^2[s(1-\gamma_2)-1][-\gamma_1-2(s-1)]}{\gamma_1^4[s(1-\gamma_2)-1]^2}, \tag{A9}$$

which is negative by (A6) and since $s>1$. Third,

$$\frac{\partial \xi}{\partial \gamma_2}=\frac{2\gamma_2[s-(1-\gamma_1)]\gamma_1^2[s(1-\gamma_2)-1]+\gamma_2^2[s-(1-\gamma_1)]\gamma_1^2 s}{\gamma_1^4[s(1-\gamma_2)-1]^2}, \tag{A10}$$

which is positive by (A6) and because $s>1$. □

Part B

This appendix analyzes the model as developed in Sections 2 and 3 of the paper while relaxing the assumption that the means of θ_1 and θ_2 respectively equal to zero. Specifically, assume that now $E\theta_1\equiv\mu_1\neq 0$ and $E\theta_2\equiv\mu_2\neq 0$, with $\mu_1\neq\mu_2$. Thus, $\sigma_1^2\equiv E(\theta_1-\mu_1)^2= E\theta_1^2-\mu_1^2$ and $\sigma_2^2\equiv E(\theta_2-\mu_2)^2= E\theta_2^2-\mu_2^2$. All other assumptions adopted in Sections 2 and 3 continue to hold.

Consider, first, decentralized (anarchic) policy-making. Jurisdiction 1 chooses p_1 to maximize (A1), giving rise to (A2). Recognizing that jurisdiction 1's equilibrium strategy is a function of θ_1 and taking expectation with respect to θ_1 on both sides of (A2) yields:

$$\gamma_1\mu_1+(1-\gamma_1)Ep_2(\theta_2)=Ep_1(\theta_1)\,. \tag{B1}$$

By symmetry,

$$\gamma_2\mu_2+(1-\gamma_2)Ep_1(\theta_1)=Ep_2(\theta_2)\,. \tag{B2}$$

Solving the systems of equations (B1) and (B2) for $Ep_1(\theta_1)$ and $Ep_2(\theta_2)$ and plugging the resulting expressions into (B1) and (B2) gives

$$p_1^D=\gamma_1\theta_1+(1-\gamma_1)\frac{\gamma_2\mu_2+(1-\gamma_2)\gamma_1\mu_1}{\gamma_1+\gamma_2-\gamma_1\gamma_2} \tag{B3}$$

$$p_2^D=\gamma_2\theta_2+(1-\gamma_2)\frac{\gamma_1\mu_1+(1-\gamma_1)\gamma_2\mu_2}{\gamma_1+\gamma_2-\gamma_1\gamma_2}\,. \tag{B4}$$

Observe that if $\mu_1=\mu_2=0$, (B3) and (B4) reduce to $p_1^D=\gamma_1\theta_1$ and $p_2^D=\gamma_2\theta_2$, as stipulated in Claim 1 in the paper. Then,

$$EW^D=E\left\{-\omega_1\gamma_1(\theta_1-p_1^D)^2-\omega_1(1-\gamma_1)(p_1^D-p_2^D)^2-\omega_2\gamma_2(\theta_2-p_2^D)^2-\omega_2(1-\gamma_2)(p_2^D-p_1^D)^2\right\}, \tag{B5}$$

where

$$\theta_1-p_1^D=(1-\gamma_1)\theta_1-(1-\gamma_1)\frac{\gamma_2\mu_2+(1-\gamma_2)\gamma_1\mu_1}{\gamma_1+\gamma_2-\gamma_1\gamma_2} \tag{B6}$$

$$\theta_2 - p_2^D = (1-\gamma_2)\theta_2 - (1-\gamma_2)\frac{\gamma_1\mu_1 + (1-\gamma_1)\gamma_2\mu_2}{\gamma_1+\gamma_2-\gamma_1\gamma_2} \tag{B7}$$

$$p_1^D - p_2^D = \gamma_1\theta_1 - \gamma_2\theta_2 + \frac{(1-\gamma_1)\gamma_2^2\mu_2 - (1-\gamma_2)\gamma_1^2\mu_1}{\gamma_1+\gamma_2-\gamma_1\gamma_2}. \tag{B8}$$

Inserting (B6)-(B8) into (B5), taking the expectation, noting that $\mathrm{E}\theta_1^2=\sigma_1^2+\mu_1^2$, $\mathrm{E}\theta_2^2=\sigma_2^2+\mu_2^2$, and collecting terms, it follows that

$$\mathrm{E}W^D = -\left[\omega_1(1-\gamma_1)\gamma_1 + \omega_2(1-\gamma_2)\gamma_1^2\right]\sigma_1^2 - \left[\omega_2(1-\gamma_2)\gamma_2 + \omega_1(1-\gamma_1)\gamma_2^2\right]\sigma_2^2 + J, \tag{B9}$$

where the term $J=J(\gamma_1,\gamma_2,\omega_1,\omega_2,\mu_1,\mu_2)$ is independent of σ_1^2 and σ_2^2. Observe that (B9) differs from the expression (3) in the paper by the term J only.

Consider, next, centralized harmonization of policymaking. Acting ex ante, before θ_1 and θ_2 are realized, the supra-jurisdictional authority chooses a uniform policy p to maximize

$$\mathrm{E}\left\{-\omega_1\gamma_1(\theta_1-p)^2 - \omega_2\gamma_2(\theta_2-p)^2\right\}. \tag{B10}$$

Obtaining and re-arranging the resulting first-order condition (second-order condition for a maximum is satisfied) gives

$$p^H = \frac{\omega_1\gamma_1\mu_1 + \omega_2\gamma_2\mu_2}{\omega_1\gamma_1 + \omega_2\gamma_2}. \tag{B11}$$

Note that if $\mu_1=\mu_2=0$, (B11) simplifies to $p^H=0$, as noted in Claim 2 in the paper. Then,

$$\mathrm{E}W^H = \mathrm{E}\left\{-\omega_1\gamma_1(\theta_1-p^H)^2 - \omega_2\gamma_2(\theta_2-p^H)^2\right\}, \tag{B12}$$

where

$$\theta_1 - p^H = \theta_1 - \frac{\omega_1\gamma_1\mu_1 + \omega_2\gamma_2\mu_2}{\omega_1\gamma_1 + \omega_2\gamma_2} \tag{B13}$$

$$\theta_2 - p^H = \theta_2 - \frac{\omega_1\gamma_1\mu_1 + \omega_2\gamma_2\mu_2}{\omega_1\gamma_1 + \omega_2\gamma_2}. \tag{B14}$$

Inserting (B13) and (B14) into (B12), taking the expectation, noting that $\mathrm{E}\theta_1^2=\sigma_1^2+\mu_1^2$, $\mathrm{E}\theta_2^2=\sigma_2^2+\mu_2^2$, and collecting terms, it follows that

$$\mathrm{E}W^H = -\omega_1\gamma_1\sigma_1^2 - \omega_2\gamma_2\sigma_2^2 + K, \tag{B15}$$

where the term $K=K(\gamma_1,\gamma_2,\omega_1,\omega_2,\mu_1,\mu_2)$ is independent of σ_1^2 and σ_2^2. Observe that (B15) and the expression (5) in the paper differ only by the term K.

Comparison of (B15) and (B9) yields

$$\Delta \equiv \mathrm{E}W^H - \mathrm{E}W^D = \nu_1\gamma_1^2\sigma_1^2 + \nu_2\gamma_2^2\sigma_2^2 + K - J, \tag{B16}$$

where $\nu_1\equiv\omega_2(1-\gamma_2)-\omega_1$ and $\nu_2\equiv\omega_1(1-\gamma_1)-\omega_2$ (see Section 3 of the paper). Letting $\mathrm{d}\sigma_1^2$, $\mathrm{d}\sigma_2^2>0$,

$$\mathrm{d}\Delta = \gamma_1^2\nu_1\mathrm{d}\sigma_1^2 + \gamma_2^2\nu_2\mathrm{d}\sigma_2^2, \tag{B17}$$

which coincides with expression (7) in the paper. Thus, when $\mu_1\neq\mu_2\neq 0$, the Proposition stated in Section 3 continues to hold. □

Part C

This appendix shows that under the assumptions of the model as specified in Sections 2 and 3 of the paper, policy harmonization emerges endogenously under the centralized, top-down approach: acting ex ante, the supra-jurisdictional authority chooses a uniform policy $p_1=p_2=p$ *even if* mandating diverse policies in different jurisdictions (i.e. $p_1 \neq p_2$) is a possibility.

Consider the following problem: acting ex ante, before θ_1 and θ_2 are realized, the supra-jurisdictional authority chooses p_1 and p_2 to maximize

$$\mathrm{E}\left\{-\omega_1\gamma_1(\theta_1 - p_1)^2 - \omega_1(1-\gamma_1)(p_1 - p_2)^2 - \omega_2\gamma_2(\theta_2 - p_2)^2 - \omega_2(1-\gamma_2)(p_2 - p_1)^2\right\} \tag{C1}$$

Obtaining and re-arranging the resulting first-order necessary conditions (second-order conditions for a maximum are satisfied) gives

$$[\omega_1 + \omega_2(1-\gamma_2)]p_1 - [\omega_1(1-\gamma_1) + \omega_2(1-\gamma_2)]p_2 = 0 \tag{C2}$$

$$[\omega_1(1-\gamma_1) + \omega_2(1-\gamma_2)]p_1 - [\omega_2 + \omega_1(1-\gamma_1)]p_2 = 0 \tag{C3}$$

Equations (C2) and (C3) constitute a linear homogenous system AX=0, where X=[p_1 p_2]' and the matrix of coefficients equals

$$\mathrm{A} = \begin{bmatrix} \omega_1 + \omega_2(1-\gamma_2) & -[\omega_1(1-\gamma_1) + \omega_2(1-\gamma_2)] \\ \omega_1(1-\gamma_1) + \omega_2(1-\gamma_2) & -[\omega_2 + \omega_1(1-\gamma_1)] \end{bmatrix}. \tag{C4}$$

From (C4), it follows that the determinant of A equals

$$\det(\mathrm{A}) = \omega_1\omega_2(\gamma_1\gamma_2 - \gamma_1 - \gamma_2) - \omega_2^2(1-\gamma_2)\gamma_2 - \omega_1^2(1-\gamma_1)\gamma_1 \neq 0\,. \tag{C5}$$

(C5) implies $p_1=p_2=0$. □