

Online Appendix to

**Mandating Behavioral Conformity in Social
Groups with Conformist Members**

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1. Summary

This Online Appendix contains a series of mathematical derivations and results supplementing the material in the main body of, and the Appendix to, the paper. The presented material is organized as follows. Section 2 explores our set-up under full information. Section 3 relaxes the assumption that the mean of θ_i 's is zero.

The main conclusions based on the analysis presented herein are: First, the comparison of group welfare under different scenarios—the central aim of our paper—is in the most general case algebraically untractable under both the full information case and when relaxing the assumption that the mean of θ_i 's is zero. In these cases, the comparison of group welfare under different scenarios is tractable only under strong assumptions regarding the extent of the group's homogeneity.

Second, the scenario when the planner mandates behavior (but not behavioral conformity) is mathematically similar under both the full information case and the incomplete information scenario in that fully characterizing the problem's solution for the most general case is analytically untractable. We do, however, state a set of sufficient conditions under which we can analytically fully characterize the solution to the planner's problem of mandating behavior (but not behavioral conformity); under those conditions, the solution to the planner's problem of mandating behavior (but not behavioral conformity) in the incomplete information case coincides with the solution to the planner's problem of mandating behavioral conformity.

2. Full Information

Key assumption: $\theta_1, \dots, \theta_n$ are common knowledge.

Two definitions, which we use in analysis below:

Defⁿ 1 (*Limited homogeneity*): $\omega_i = \omega$, $\lambda_i = \lambda$ for all i .

Defⁿ 2 (*Full homogeneity*): $\omega_i = \omega$, $\lambda_i = \lambda$, $\theta_i = \theta$ for all i .

2.1 Non-Cooperative Equilibrium (N)

Individuals play a static game with full information. The equilibrium concept is Nash equilibrium. To find reaction function of individual i , we maximize U_i (expression (1) in the paper) choosing a_i , which implies the following first-order condition:

$$(S1) \quad \frac{\partial U_i}{\partial a_i} = 2\lambda_i(\theta_i - a_i) - 2(1 - \lambda_i)\left(a_i - \frac{1}{n-1} \sum_{j \neq i} a_j\right) = 0.$$

After re-arranging terms in (S1), we obtain

$$(S2) \quad a_i - \frac{1 - \lambda_i}{n-1} \sum_{j \neq i} a_j = \lambda_i \theta_i.$$

The system of equations (S2) for $i=1, \dots, n$ has a unique solution (a_1^N, \dots, a_n^N) because the matrix of coefficients is a dominant diagonal matrix, and hence non-singular. (The matrix of coefficients is identical to the one featured in expression (A5) of the paper.) Thus, under full information, we have a unique Nash equilibrium in the non-cooperative scenario. From (S2), however, it is clear that (a_1^N, \dots, a_n^N) depends in a complicated way on λ_i 's and θ_i 's, which, as we demonstrate below (see Section 2.4), unfortunately renders welfare analysis in the general case untractable.

Under 'limited homogeneity' the system (S2) for $i=1, \dots, n$ becomes

$$(S3) \quad a_i - \frac{1 - \lambda}{n-1} \sum_{j \neq i} a_j = \lambda \theta_i$$

for $i=1, \dots, n$. Adding up equations (S3) for $i=1, \dots, n$ and simplifying we obtain

$$(S4) \quad \sum_i a_i = \sum_i \theta_i.$$

Next, multiply LHS and RHS of (S4) by $(1-\lambda)/(n-1)$ to obtain

$$(S5) \quad \frac{1-\lambda}{n-1} a_i + \frac{1-\lambda}{n-1} \sum_{j \neq i} a_j = \frac{1-\lambda}{n-1} \sum_j \theta_j.$$

Adding up (S3) and (S5) and solving for a_i gives

$$(S6) \quad a_i^N = \frac{(n-1)\lambda}{n-\lambda} \theta_i + \frac{1-\lambda}{n-\lambda} \sum_j \theta_j.$$

Under 'full homogeneity', we thus obtain (using (S6)) $a_i^N = \theta$.

2.2 Mandating Behavioral Conformity (MBC)

The social planner, knowing all ω_i 's, λ_i 's, and θ_i 's, chooses the group-wide common action a to maximize $\sum_i \omega_i U_i = -\sum_i \omega_i \lambda_i (\theta_i - a)^2$. Obtaining the corresponding first-order condition and re-arranging terms, we have

$$(S7) \quad a^{MBC} = \sum_i \frac{\omega_i \lambda_i}{\sum_j \omega_j \lambda_j} \theta_i.$$

Then, under 'limited homogeneity', we have $a^{MBC} = \frac{1}{n} \sum_i \theta_i \equiv \bar{\theta}$ and under 'full homogeneity', $a^{MBC} = \theta$.

2.3 Mandating Behavior (but not Behavioral Conformity) (MB)

We define 'mandating behavior' as scenario where the social planner, knowing all ω_i 's, λ_i 's, and θ_i 's, chooses a vector of actions (a_1, \dots, a_n) to maximize group welfare $W \equiv \sum_i \omega_i U_i$, where U_i is defined in expression (1) in the paper. Note that this is the first-best solution from group welfare point of view. Having an understanding of the algebra of this problem, however, is instructive for our analysis in Section 3 of this Online Appendix.

The first-order conditions for 'mandating behavior' are:

$$(S8) \quad \frac{\partial W}{\partial a_i} = \omega_i \frac{\partial U_i}{\partial a_i} + \sum_{j \neq i} \omega_j \frac{\partial U_j}{\partial a_i} = 0, \quad i = 1, \dots, n,$$

where

$$(S9) \quad \frac{\partial U_i}{\partial a_i} = 2\lambda_i(\theta_i - a_i) - 2(1 - \lambda_i)(a_i - \bar{a}_{-i}),$$

$$(S10) \quad \frac{\partial U_j}{\partial a_i} = 2 \frac{1 - \lambda_j}{n - 1} (a_j - \bar{a}_{-j}).$$

Plugging (S9) and (S10) into (S8) gives

$$(S11) \quad 2\omega_i[\lambda_i(\theta_i - a_i) - (1 - \lambda_i)(a_i - \bar{a}_{-i})] + 2 \sum_{j \neq i} \omega_j \frac{1 - \lambda_j}{n - 1} (a_j - \bar{a}_{-j}) = 0,$$

which, after simplifying, yields:

$$(S12) \quad \begin{aligned} & -\omega_i a_i + \omega_i \lambda_i \theta_i + \omega_i \frac{1 - \lambda_i}{n - 1} \sum_{j \neq i} a_j \\ & + \frac{1}{n - 1} \sum_{j \neq i} \omega_j (1 - \lambda_j) a_j - \frac{1}{(n - 1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j) \sum_{k \neq j} a_k = 0. \end{aligned}$$

We can re-write the last part of LHS of (S12) as

$$(S13) \quad \frac{1}{(n - 1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j) \sum_{k \neq j} a_k = \left[\frac{1}{(n - 1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j) \right] a_i + \sum_{j \neq i} \left[\frac{1}{(n - 1)^2} \sum_{\substack{k \neq i \\ k \neq j}} \omega_k (1 - \lambda_k) \right] a_j.$$

Multiplying (S12) by (-1) and using (S13) then gives

$$(S14) \quad \left[\omega_i + \frac{1}{(n-1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j) \right] a_i - \sum_{j \neq i} \left[\omega_i \frac{1 - \lambda_i}{n-1} + \omega_j \frac{1 - \lambda_j}{n-1} - \frac{1}{(n-1)^2} \sum_{\substack{k \neq i \\ k \neq j}} \omega_k (1 - \lambda_k) \right] a_j = \omega_i \lambda_i \theta_i.$$

To ease the exposition, let's define:

$$(S15) \quad b_{ii} = \omega_i + \frac{1}{(n-1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j)$$

and

$$(S16) \quad b_{ij} = - \left[\omega_i \frac{1 - \lambda_i}{n-1} + \omega_j \frac{1 - \lambda_j}{n-1} - \frac{1}{(n-1)^2} \sum_{\substack{k \neq i \\ k \neq j}} \omega_k (1 - \lambda_k) \right]$$

so that (S14) can be written as

$$(S17) \quad b_{ii} a_i + \sum_{j \neq i} b_{ij} a_j = \omega_i \lambda_i \theta_i.$$

From (S15), observe that b_{ii} is strictly positive. The sign of b_{ij} , on the other hand, is ambiguous (see (S16)).

We would like to characterize the solution to the system (S17) for $i=1, \dots, n$. In particular, we would like to know if the solution exists, and, if it exists, whether it is unique. To do so, we would like to verify or refute non-singularity of the matrix of coefficients of system (S17) for $i=1, \dots, n$. Let us call this matrix $B=[b_{ij}]_{n \times n}$. To check non-singularity of B , we try to verify if the matrix of coefficients is dominant diagonal, which is true if $|b_{ii}| > \sum_{j \neq i} |b_{ij}|$ for all i .

As it turns out, the matrix B is *not* dominant diagonal for all possible parameter values. To see this, take the example of $n=3$, $\lambda_1=\lambda_2=0.9$, $\lambda_3=0.1$, $\omega_1=\omega_2=0.1$, $\omega_3=0.8$, in which case $|b_{11}|-|b_{12}|-|b_{13}|=-0.25 < 0$. We also know that if a matrix is not dominant diagonal, it can still be non-singular. (For the example of $n=3$, $\lambda_1=\lambda_2=0.9$, $\lambda_3=0.1$, $\omega_1=\omega_2=0.1$, $\omega_3=0.8$, the matrix B is indeed non-singular: using Mathematica, we verified that the determinant of the matrix equals 0.0114131.)

We have been unable to analytically verify or refute non-singularity of the matrix B in the most general case. (Working with the case for $n=3$, we tried to check whether the determinant of B is ever zero for any of the permissible parameter values $\{(\lambda_1, \lambda_2, \lambda_3, \omega_1, \omega_2, \omega_3): \lambda_i \in (0, 1), \omega_i > 0 \text{ for all } i \in G\}$. A numerical exploration that made use of Mathematica's commands such as FindMinimum and NMinimize did not indicate that the determinant of B is equal to zero for any of the permissible parameter values; the determinant of B seemed to always be positive. This, however, is clearly not an analytically-grounded statement and cannot be generalized to the most general case for arbitrary n .)

As a result, we characterize the solution to the system (S17) for $i=1, \dots, n$ by finding sufficient conditions such that matrix B is dominant diagonal and, hence, non-singular. In particular, we have the following result:

Result: The matrix of the coefficients of the system defined by (S17) for $i=1, \dots, n$ is dominant diagonal, and, hence, non-singular, if $b_{ij} < 0$ for all i and $j \neq i$, which in turn holds if $\lambda_i = \lambda_j$ and $\omega_i = \omega_j$ for all i, j .

Proof: To prove this Result, suppose that b_{ij} , defined in (S16), is negative for all i and $j \neq i$. Then,

$$\begin{aligned} \sum_{j \neq i} |b_{ij}| &= \sum_{j \neq i} \left[\omega_i \frac{1 - \lambda_i}{n-1} + \omega_j \frac{1 - \lambda_j}{n-1} - \frac{1}{(n-1)^2} \sum_{\substack{k \neq i \\ k \neq j}} \omega_k (1 - \lambda_k) \right] \\ &= \omega_i (1 - \lambda_i) + \sum_{j \neq i} \omega_j (1 - \lambda_j) \left[\frac{1}{n-1} - \frac{n-2}{(n-1)^2} \right] \\ &= \omega_i (1 - \lambda_i) + \frac{1}{(n-1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j), \end{aligned}$$

which is smaller than $|b_{ii}| = b_{ii}$ defined in (S15). Hence, the resulting matrix of coefficients of the system defined by (S17) for $i=1, \dots, n$ is dominant diagonal, therefore non-singular, and, thus, the solution to the system defined by (S17) for $i=1, \dots, n$, which we denote as $(a_1^{MB}, \dots, a_n^{MB})$, is unique. Note that $\omega_i = \omega$ and $\lambda_i = \lambda$ for all i implies that $b_{ij} = -[\omega(1-\lambda)n]/(n-1)^2 < 0$, which proves the last part of Result. ■

Two remarks are in place. First, b_{ij} in general can be positive. For example, if λ_i and λ_j are close to 1 and all other λ 's are close to 0 (that is, the social group can be describe as

heterogeneous with respect to the strength of members' innate conformist tendencies), then, from (S16), $b_{ij} > 0$. Second, when it is unique, the closed-form solution to the problem of 'mandating behavior' is evidently algebraically very messy.

In contrast, we are able to obtain more tractable closed-form solutions to the problem of 'mandating behavior' using Definitions 1 and 2. Under 'limited homogeneity', (S14) becomes

$$(S19) \quad \frac{n-\lambda}{n-1} a_i - \frac{(1-\lambda)n}{(n-1)^2} \sum_{j \neq i} a_j = \lambda \theta_i.$$

Summing up expressions (S19) for $i=1, \dots, n$, we obtain

$$(S20) \quad \sum_i a_i = \sum_i \theta_i.$$

Multiplying LHS and RHS of (S20) by $(1-\lambda)n/(1-n)^2$ and adding up the resulting expression and expression (S19) gives

$$(S21) \quad \frac{n^2 - 2n\lambda + \lambda}{(n-1)^2} a_i = \lambda \theta_i + \frac{(1-\lambda)n}{(n-1)^2} \sum_j \theta_j.$$

Expression (S21) can thus be used to solve for the optimal value of a_i :

$$(S22) \quad a_i^{MB} = \frac{1}{n^2 - 2n\lambda + \lambda} \left[(n-1)^2 \lambda \theta_i + (1-\lambda)n \sum_j \theta_j \right].$$

From (S22), it follows that under 'full homogeneity' $a_i^{MB} = \theta$.

2.4 Welfare Analysis

We know that 'mandating behavior' coincides with the first-best solution from the group welfare point of view. It is therefore sensible only to attempt to compare group welfare under 'mandating behavioral conformity' and under the non-cooperative equilibrium. From (S2), it is clear that (a_1^N, \dots, a_n^N) depends in a complicated way on λ_i 's and θ_i 's. As a result, the comparison of group welfare under the non-cooperative scenario with that under mandated behavioral conformity is in general algebraically untractable.

In fact, the algebra of group welfare comparison under the two scenarios is untractable even under 'limited homogeneity', in which case group welfare under mandated behavioral conformity equals (see (S7))

$$(S23) \quad W^{MBC} = -\omega \lambda \sum_i (\theta_i - \bar{\theta}_i)^2,$$

and group welfare under the non-cooperative scenario can be shown (using (S6)) to equal

$$(S24) \quad W^N = -\omega\lambda \sum_i \left[\frac{(1-\lambda)(n-1)}{n-\lambda} \theta_i - \frac{1-\lambda}{n-\lambda} \sum_{j \neq i} \theta_j \right]^2 - \omega(1-\lambda) \sum_i \left[\frac{(n-1)\lambda}{n-\lambda} \theta_i - \frac{\lambda}{n-\lambda} \sum_{j \neq i} \theta_j \right]^2$$

Evidently, comparison of (S23) and (S24) is analytically untractable. In contrast, under 'full homogeneity', $a_i^N = a^{MBC} = \theta$, and, as a result, $W^N = W^{MBC}$ trivially.

3. Relaxing the assumption $E\theta_i=0$ for all i

Key assumption: $E\theta_i = \mu_i$.

We introduce another two definitions, which we use in the analysis below:

Defⁿ 3 (*Limited ex-ante homogeneity*): $\mu_i = \mu$ for all i

Defⁿ 4 (*Full ex-ante homogeneity*): $\omega_i = \omega$, $\lambda_i = \lambda$, $\mu_i = \mu$ for all i .

3.1 Non-Cooperative Equilibrium (N)

Group members play a Bayesian-Nash game. To find the non-cooperative equilibrium, follow the steps outlined in Proof of Lemma in the Appendix of the paper. Under the assumption that $E\theta_i = \mu_i$, expression (A4) becomes

$$(S25) \quad Ea_i(\theta_i) - (1-\lambda_i)(n-1)^{-1} \left[\sum_{j \neq i} Ea_j(\theta_j) \right] = \lambda_i \mu_i.$$

Upon comparison of (S25) with (A4), it is clear that they differ only in terms of the RHS. Thus, the matrix of the coefficients implied by system (S25) for $i=1, \dots, n$ is dominant diagonal and, thus, non-singular. Therefore, there exists a unique vector $(Ea_1^*(\theta_1), \dots, Ea_n^*(\theta_n))$, which solves the system (S25) for $i=1, \dots, n$ and depends in a complicated way on λ_i 's and μ_i 's. As a result, a_i^N , as implied by (A3), no longer equals $\lambda_i \theta_i$, but rather equals

$$(S26) \quad a_i(\theta_i) = \lambda_i \theta_i + (1-\lambda_i)(n-1)^{-1} \left[\sum_{j \neq i} Ea_j^*(\theta_j) \right],$$

an expression containing $\{\lambda_j\}_{j \neq i}$ and μ_i 's.

Expression (S26) is not significantly simplified even if we assume 'limited ex-ante homogeneity', in which case (S25) becomes

$$(S27) \quad Ea_i(\theta_i) - (1-\lambda_i)(n-1)^{-1} \left[\sum_{j \neq i} Ea_j(\theta_j) \right] = \lambda_i \mu.$$

Again, it is clear that the vector $(Ea_1^*(\theta_1), \dots, Ea_n^*(\theta_n))$, and thus equilibrium a_i^N 's (see (A3)), depend in a complicated way on λ_i 's and μ .

In contrast, under 'full ex-ante homogeneity', (S27) simplifies to

$$(S28) \quad Ea_i(\theta_i) - (1-\lambda)(n-1)^{-1} \left[\sum_{j \neq i} Ea_j(\theta_j) \right] = \lambda\mu.$$

Observe that the system (S28) for $i=1, \dots, n$ is algebraically identical to the system (S3) for $i=1, \dots, n$ if in (S3) we replace a_i with $Ea_i(\theta_i)$, a_j with $Ea_j(\theta_j)$, and θ_i with μ . Hence, using steps analogous to (S4)-(S6), it follows that $Ea_i(\theta_i) = \mu$ and, using (S26), we are able to obtain a tractable closed-form solution for a_i^N equal to

$$(S29) \quad a_i^N = \lambda\theta_i + (1-\lambda)\mu.$$

3.2 Mandating Behavioral Conformity (MBC)

The group planner chooses a to maximize $E[-\sum_i \omega_i \lambda_i (\theta_i - a)^2]$. The resulting first-order condition implies $\sum_i \omega_i \lambda_i (\mu_i - a) = 0$, which in turn implies

$$(S30) \quad a^{MBC} = \sum_i \frac{\omega_i \lambda_i}{\sum_j \omega_j \lambda_j} \mu_i.$$

From (S30), it follows that under both 'limited ex-ante homogeneity' and 'full ex-ante homogeneity', therefore, $a^{MBC} = \mu$.

3.3 Mandating Behavior (but not Behavioral Conformity) (MB)

The social planner, knowing all ω_i 's and λ_i 's, but not knowing exact realizations of θ_i 's, chooses a vector of actions (a_1, \dots, a_n) to maximize expected group welfare $EW = E\sum_i \omega_i U_i$, where U_i is

defined in expression (1) in the paper. The first-order conditions are $E\left[\frac{\partial W}{\partial a_i}\right] = 0$ for $i=1, \dots, n$.

Using the same steps as in Section 2.3 of this Online Appendix and taking expectation, we obtain:

$$(S31) \quad E\left[\frac{\partial W}{\partial a_i}\right] = 2\omega_i[\lambda_i(\mu_i - a_i) - (1-\lambda_i)(a_i - \bar{a}_{-i})] + 2\sum_{j \neq i} \omega_j \frac{1-\lambda_j}{n-1} (a_j - \bar{a}_{-j}) = 0.$$

Observe that equation (S31) is algebraically identical to equation (S11) if in (S11) we replace θ_i by μ_i . Therefore, the analysis of the system of equations (S31) for $i=1, \dots, n$ is algebraically

identical to that of the system (S11) for $i=1, \dots, n$. We can therefore immediately state the crucial expression, which is an algebraic equivalent of expression (S14) with θ_i replaced by μ_i :

$$(S32) \quad \left[\omega_i + \frac{1}{(n-1)^2} \sum_{j \neq i} \omega_j (1 - \lambda_j) \right] a_i - \sum_{j \neq i} \left[\omega_i \frac{1 - \lambda_i}{n-1} + \omega_j \frac{1 - \lambda_j}{n-1} - \frac{1}{(n-1)^2} \sum_{\substack{k \neq i \\ k \neq j}} \omega_k (1 - \lambda_k) \right] a_j = \omega_i \lambda_i \mu_i.$$

From the analysis in Section 2.3 in this Online Appendix, we know that if

$$(S33) \quad \omega_i \frac{1 - \lambda_i}{n-1} + \omega_j \frac{1 - \lambda_j}{n-1} - \frac{1}{(n-1)^2} \sum_{\substack{k \neq i \\ k \neq j}} \omega_k (1 - \lambda_k) > 0$$

for all i and $j \neq i$, then the matrix of coefficients of the system (S32) for $i=1, \dots, n$ is dominant diagonal and hence non-singular, and the system has a unique—albeit algebraically messy—solution. Note in particular that if $\mu_i=0$ for all i , as assumed in the paper, and (S33) holds, then the unique solution is $a_i=0$ for all i , that is, mandating behavior (but not conformity) implies 'mandating behavioral conformity'.

Note that the system (S32) for $i=1, \dots, n$ is not much simpler even if we assume 'limited ex-ante homogeneity' when $\mu \neq 0$ (in the sense that it does not allow for tractable closed-form solution). On the other hand, 'full ex-ante homogeneity' implies, using analogous steps as those in Section 2.3 of this Online Appendix, that $a_i^{MB} = \mu$.

3.4 Welfare Analysis

Given analysis in Sections 3.1-3.3 of this Online Appendix, it is clear that while the model can still be solved when relaxing the assumption $E\theta_i=0$ for all i , the welfare analysis—the central aim of this paper—becomes untractable. Algebraically, the reason is that in the most general case, assuming either $E\theta_i=\mu_i$ or $E\theta_i=\mu$ results in a non-cooperative equilibrium action, which depends on λ_i 's and μ_i 's (or μ) and a similarly untractable closed-form solution in the cases of 'mandating behavior' (but not behavioral conformity) and 'mandating behavioral conformity'.

Welfare analysis is in fact untractable even if we assume 'limited ex-ante homogeneity' (Definition 3). In contrast, 'full ex-ante homogeneity' does allow for tractability. Mandating

behavior (*MB*) and mandating behavioral conformity (*MBC*) leads to $a_i^{MB}=a_i^{MBC}=\mu$. Therefore, the expected group welfare under the two regimes equals

$$(S34) \quad EW^{MB} = EW^{MBC} \equiv EW^M = -\omega\lambda \sum_i E(\theta_i - \mu)^2 = -n\omega\lambda .$$

In contrast, under the non-cooperative scenario, we have (see (S29)) $a_i^N = \lambda\theta_i + (1-\lambda)\mu$. Therefore, evaluated at (a_1^N, \dots, a_n^N) , $E(\theta_i - a_i)^2 = (1-\lambda)^2$ and $E(a_i - \bar{a}_{-i})^2 = \lambda^2 \frac{n}{n-1}$ so that

$$EU_i = -(1-\lambda)\lambda \frac{n-1+\lambda}{n-1} \text{ and therefore}$$

$$(S35) \quad EW^N = \sum_i \omega_i EU_i = -n\omega(1-\lambda)\lambda \frac{n-1+\lambda}{n-1}.$$

Thus, to compare EW^M and EW^N , calculate

$$(S36) \quad \frac{EW^N}{EW^M} = (1-\lambda) \frac{n-1+\lambda}{n-1}.$$

We next check whether the RHS of (S36) is greater or smaller than 1:

$$(S37) \quad (1-\lambda) \frac{n-1+\lambda}{n-1} - 1 = \frac{\lambda(2-n-\lambda)}{n-1}.$$

It is straightforward to see that (S37) is always negative for any $n \geq 2$ and $\lambda \in (0,1)$: If $n=2$, then (S37) equals $-\lambda^2 < 0$. If $n \geq 3$, then the numerator of (S37) is a quadratic function of λ with roots 0 and $2-n$, implying that for $\lambda \in (0,1)$ this function is negative.

From (S36), we thus have $EW^N/EW^M < 1$, which, because $EW^M < 0$ (see (S34)), implies that $EW^N > EW^M$.